

High School Math

What should my students learn in math each year?

All students should reach the expectations outlined in the NYS standards. This means that no matter what curricular resources your school uses, there are certain experiences all students in high school. This learning map helps you know what your students should be learning and details examples of research validated pedagogical practices that you can employ to create access to rich and culturally responsive grade level content.

<p>Math</p> <p>Algebra I</p> <p>Fall Semester</p>	<p>By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Algebra I is the first mathematics course in high school and the focal point is functions; specifically, linear, quadratic, and exponential functions. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of equations and inequalities and make conjectures about the form that an equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function. (NY NGLS, 2017)</p> <p><i>The Domains</i></p> <p>The Next Generation Math Standards are organized on large groups of related standards called domains. Domains are not curriculum units, but link standards together by grouping them around big ideas that are revisited at different point across the course. The order in which the standards and domains are presented and organized <u>is not</u> the order in which they need to be taught (they are interconnected). Educators and curriculum designers need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated. (NY NGLS, 2017)</p> <p>Special note for blended and remote instruction:</p> <p>As we move into the fall, we know that there will be a need for digital resources that support blended and remote learning to support the schools shared and inclusive digital curriculum. Linked throughout this learning map are free, digital resources that support the learning that occurs in the beginning of 9th grade such as 3 Act Math Tasks, Desmos, and MARS lessons. These digital resources may be used by teachers to improve students' experience as they interact with the content and enhance existing resources in their shared, inclusive and digital curriculum. We ask that you continue to provide ongoing opportunities for students to interact with the digital resources and tools as they practice these skills, whether in-person or remote learning setting.</p> <p>How to Use This Learning Map</p> <p>Below you will find the experiences that students typically have during the fall semester of Algebra I and the learning that should takes place. To create this learning map, the design team considered the most used curricula across the NYC DOE, (e.g., Eureka/EngageNY) alongside the Priority Learning Standards in Mathematics. Although this document is completely aligned to the NYS Next Generation Learning Standards (NGLS), the language used is not an exact match, but rather a description of what the learning experience of the fall semester should look like.</p>
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	<p>In addition to expected learning experiences, this learning map identifies research-validated pedagogical practices that teachers can employ to create access to rich, culturally responsive grade level content. The pedagogical practices suggested in the last column are not meant to be a one-to-one correspondence to the descriptions of learning experiences of the left column.</p> <p>While using this learning map, it is important to keep in mind that the instructional sequence of one’s school curriculum is carefully and intentionally designed to maintain program fidelity. Lesson omissions or modifications of the order of the curriculum sequence should be carefully considered as it may have unintended and adverse impact on students’ current and future acquisition of mathematical competencies.</p>	
Domains	What will the learning look like? <i>In the fall semester, students in Algebra have experiences that support the learning below.</i>	What pedagogical practices can support this? <i>Practices that create access to rich, culturally responsive grade -level work include but are not limited to the examples below.</i>
Number and Quantity -The Real Number System Creating Equations Seeing Structure in Expressions Reasoning with Equations and Inequalities	<p>Students are provided with opportunities to:</p> <ul style="list-style-type: none"> introduce functions as they arise from real life situations that have to do with change. use graphs to reason abstractly and quantitatively with representations of linear, quadratic and exponential relationships choose and interpret units to solve problems related to the graphs that represent real life situations. understand how these mathematical properties (commutative, associative, and distributive properties) can be used to see and express complex algebraic equivalencies. solve linear and nonlinear equations (linear, quadratic, exponential) and systems of equations and inequalities, and explain their reasoning for choosing a specific strategy (i.e., graphical, substitution or elimination). solve word problems involving systems of equations and inequalities. 	<p>Teachers may:</p> <ul style="list-style-type: none"> Support student language development by contrasting the term “function” as it is used in everyday language with its discipline-specific meaning. [i.e., object-level change of language, Sfard (2012)] Engage students in making connections among mathematical representations to foster their ability to see structure and recognize equivalencies among algebraic expressions (Principle to Action, op 24-28) Connecting representations routines for reasoning support students in making connections. Provide students with opportunities to engage in a cognitively demanding task that allow for multiple entry-points for students grapple with the complexity of solving problems (i.e., the productive struggle) and provide appropriate scaffolds that support students learning without degrading the rigor of the task (e.g., prompts, feedback, info graphs). (Principle to Action, pp 48-52) Help students move from “surface” to “deep” learning by providing opportunities for making explicit observations (noticing from students) about the relationship between linear and non-linear relationships and helping students make connections between these two concepts. Routines are structures that allow students to make these connections from their prior knowledge (e.g. number talks, guided questions, worked examples, routines for reasoning) and then follow these new connections with the associated procedures. (Visible Learning for Mathematics, pp 100 – 114)

<p>Statistics and Probability Interpreting Categorical and Quantitative Data</p>	<ul style="list-style-type: none"> develop a set of tools for understanding and interpreting variability in data and begin to make informed decisions about data understand the relationship between the shape of the distribution (possibly represented as verbal description or graphically in a histogram or boxplot) and the appropriate descriptions of measures of center and spread. explore positive and negative linear relationships and use the correlations coefficient to describe the strength and direction of the linear relationship 	<ul style="list-style-type: none"> Student conceptual knowledge development cannot be separated from language development (i.e., language mediates content). For example, statistical thinking requires the development of academic language for new concepts (e.g., variability, frequency, inter-quartile range, correlation-coefficient, bivariate). Teachers should consider providing opportunities for oral and written discourse that facilitates their concept and language development concurrently (i.e., meta-level change of language). (Sfard, 2012). Engage students with a set of mathematical texts (e.g., expository text, examples and non-examples, explanations, arguments, open-ended tasks, diagrams, videos, podcast, etc.) that include essential knowledge on these topics (Teaching Advanced Literacy Skill, pp 53-55) Promote oral discourse by providing purposeful questions that help students draw knowledge from a set of texts and their learning experiences. (Teaching Advanced Literacy Skill, pp 56-59)
<p>Functions -Linear, Quadratic, and Exponential Models</p>	<ul style="list-style-type: none"> analyze bivariate data using a two-way frequency table and relative frequency tables formalize their understanding of functions to include the concepts of domain and range as well as function notation. (Note: some curricula choose to build this from students understanding of arithmetic and geometric sequences). 	<ul style="list-style-type: none"> Build student’s schema by providing students opportunities to use prior knowledge to reason and problem solve. This provides teachers opportunities to assess student thinking and provide feedback that builds on prior knowledge. For example, use existing understanding of rules that establish a correspondence among set of numbers to rules that restrict the domains and range. (Principles to Actions, pp 17-18) Promote student agency of their own learning by allowing them to author the contextual situation (from their own life experiences) that can be modeled by mathematical functions. (Visible Learning for Teachers, pp 91-99) (click here for more)
<p>Functions - Interpreting Functions</p>	<ul style="list-style-type: none"> understand the distinctions between linear and exponential functions and between additive and multiplicative change. Interpret different representations of these functions and relationships (e.g., graphically, symbolically, verbally) apply their understanding of functions to abstract and contextual situations and describe key features of the function (i.e., recognizing, evaluating and interpreting functions). 	<p>Digital Tools & Resources</p> <ul style="list-style-type: none"> Virtual dice: the probabilities of different numbers obtained by the throw of two dice can offer a good introduction to the ideas of probability topics, which are part of the Algebra course (Click here) Virtual balance scale: this tool to strengthen understanding and computation of numerical expressions and equality. In understanding equality, one of the first things students must realize is that equality is a relationship, not an operation (Click here) Desmos: this tool is an online dynamic geometry software that includes graphic and scientific calculator and featured activities connected to linear, quadratic and exponential functions, inequalities, geometric transformations and more. (click here)

<p>Functions -Building Functions</p>	<ul style="list-style-type: none"> • explore special cases of functions (i.e., absolute value functions, step functions, and other piecewise functions) and apply this knowledge in their analysis of these special cases of functions. • understanding these special-cases of functions will also support solving complex solutions to equations in one variable. • create equations and functions to model situations and examine key features of graphs of functions, relating those features to the context of the situation 	<ul style="list-style-type: none"> • Phet Interactive Simulations: provides simulations that can be linked to the curriculum such as area models, balancing equations, fractions, trigonometry and more (click here) • Padlet: Makes the remote experience come alive with various gallery walls to choose from such as timeline, map, shelves, stream in order to display student work, build on or connect ideas, facilitate a conversation and give feedback (click here) • PBS Learning Media: high-quality resources such as short videos(including videos of real world applications of mathematics from NOVA), lessons and interactive games organized by math topics (click here) • The 3-Act Problems: Dan Meyer’s has created a spreadsheet listing “Three Acts Tasks”: (1) Act One introduces a central conflict of the story/task clearly, visually, viscerally using as few words as possible. (2) In Act Two the student (protagonist of the story) overcomes obstacles, looks for resources, and develops new tools. (3) In Act Three, the student resolves the conflict and set up a sequel/extension. (Click here) • MARS: well-engineered tools for formative and summative assessment that expose students’ mathematical knowledge and reasoning, helping teachers guide them towards improvement and monitor progress. The tools are relevant to any curriculum that seeks to deepen students' understanding of mathematical concepts and develop their ability to apply that knowledge to non-routine problems (Click here)
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Math

Geometry

Fall Semester

Over the years, students develop an understanding of the attributes and relationships of two- and three-dimensional geometric shapes that can be applied in diverse contexts. However, it is in high school ‘Geometry’ where students formalize their geometry experiences from elementary and middle school, using more precise definitions to establish the validity of geometric conjectures through deduction, proof, or mathematical arguments. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformations. Fundamental are the rigid motions: translations, rotations, reflections, and sequences of these, all of which are here assumed to preserve distance and angle measure. Reflections and rotations each explain a particular type of symmetry leading to insight into a figure’s attributes.

The Domains

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How to Use This Learning Map

Below you will find the experiences that students typically have during the fall semester of Geometry and the learning that should takes place. To create this learning map, the design team considered the most used curricula across the NYC DOE, (e.g., Eureka/EngageNY) alongside the [Priority Learning Standards in Mathematics](#). Although this document is completely aligned to the NYS Next Generation Learning Standards (NGLS), the language used is not an exact match, but rather a description of what the learning experience of the fall semester should look like.

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	<ul style="list-style-type: none"> • extend their understanding of dilation to real-world application of drawing maps to scale (i.e., properties of dilation, splitter and dilation theorems). • formalize their knowledge of Pythagorean theorem as well as devise and write a formal prove. • be formally Introduced to the ratios sine, cosine, tangent and use them to solve word problems. • extend their knowledge of trigonometry ratios to include the laws of sine and cosine (i.e., moving beyond right triangles) 	<p><u>Digital Tools & Resources</u></p> <ul style="list-style-type: none"> • GeoBoard: this manipulative can support exploration of fundamental concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons (Click here) • Virtual polygon tiles: this virtual manipulatives can support students visually see patterns and conceptualize mathematical representations as well as important abstract relationships that are central to learning geometry (Click here) • Desmos: this tool is an online dynamic geometry software that includes graphic and scientific calculator and featured activities connected to linear, quadratic and exponential functions, inequalities, geometric transformations and more. (click here) • Padlet: Makes the remote experience come alive with various gallery walls to choose from such as timeline, map, shelves, stream in order to display student work, build on or connect ideas, facilitate a conversations and give feedback (click here) • PBS Learning Media: high-quality resources such as short videos(including videos of real world applications of mathematics from NOVA), lessons and interactive games organized by math topics (click here)
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Math

Algebra II

Fall Semester

Algebra II is the capstone course of the three high school mathematics courses and is a continuation and extension of the two courses that precede it. Building on their work with linear, quadratic, and exponential functions in Algebra I, students in Algebra II extend their repertoire of functions to include polynomial, rational, radical, and trigonometric functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers, and solving exponential equations using the properties of logarithms.

Students connect the structure inherent in multi-digit whole number multiplication with multiplication of polynomials and similarly connect division of polynomials with long division of integers. Students analyze the key features of functions and their representations and relate those features back to the two quantities the function is modeling. Students synthesize and generalize what they have learned about a variety of function families. They explore (with appropriate tools) the effects of transformations on graphs of diverse functions, including functions arising in an application.

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, observational studies, and experiments. Using simulation, randomization, and careful design, students make inferences, justify conclusions, and critique statistical claims. Students create theoretical and experimental probability models following the modeling cycle. They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability. (NY NGLS, 2017)

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How to Use This Learning Map

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Domains	<p style="text-align: center;">What will the learning look like? <i>In the fall semester, students in Algebra II have experiences that support the learning below.</i></p>	<p style="text-align: center;">What pedagogical practices can support this? <i>Practices that create access to rich, culturally responsive grade -level work include but are not limited to the examples below.</i></p>
<p>Arithmetic with Polynomials and Rational Expressions</p> <p>Seeing Structure in Expressions</p>	<ul style="list-style-type: none"> • write polynomial expressions by examining successive differences and reasoning about patterns and relationships • multiply polynomial expressions using the distributive property and relate it to previous understandings, including the area model. • divide polynomials and make generalizations about polynomial expressions and their factors. • investigate patterns of special products of polynomials [e.g. $(x-a)(x^2+2a+a^2)$, $(x+a)(x-a)$] and use the structure of these patterns in applications. • develop fluency with factoring polynomials • use structures of polynomial expressions to: (1) factor special cases (e.g., difference of squares, completing the square, higher than second degree polynomials, polynomials with remainder), (2) construct their graph and (3) solve word problems (i.e., modeling with polynomials). 	<ul style="list-style-type: none"> • Mathematical discourse combines language, symbols, and visual images that are used to create meaning and construct knowledge (i.e. multisemiotic). To master algebraic thinking students must resolve how these three resources interact with each other and mathematical knowledge. Therefore, it is strongly recommended that math teachers explicitly teach the language found in mathematical discourse: <ul style="list-style-type: none"> ○ draw attention to interpretations encoded in mathematical symbolism and visual display ○ model the interactions of math symbols and language in their own discourse and text-sets ○ expect students to always construct meaning by combining these three resources ○ create a system of peer-feedback that foster meaning making. For example, student regularly use sentence frames to provide to feedback to each other and to the teachers: <ul style="list-style-type: none"> ▫ <i>"I heard you say ___ and I would like to add ___"</i>. ▫ <i>"the interpretation you offer about _____ directly contradicts _____, perhaps if we re-state it in this way _____ it would help me understand."</i> (Mathematical Discourse; O'Halloran, pp 189-211) • Metacognitive approach to learning mathematics has been repeatedly validated by research as a very power strategy for learning mathematics and academic language. Promote a metacognitive approach to learning mathematics by: <ul style="list-style-type: none"> ○ providing self-regulated prompts ("ask-yourself" questions) ○ model how to 'make a plan' for approaching a task ○ develop tools for self-assessment (e.g., rubrics, reflection prompts) ○ provide tools that facilitate a perception of progress (e.g., benchmarks, finished model).

<p>Arithmetic with Polynomials and Rational Expressions</p> <p>Reasoning with Equations and Inequalities</p> <p>Number and Quantity -The Complex Number System</p> <p>Functions - Trigonometric Functions</p>	<ul style="list-style-type: none"> • rewrite polynomial expressions in equivalent forms and compare rational expressions by analyzing numerically, graphically and algebraically. • rewrite rational expressions into equivalent forms using properties of operations and connecting to properties of operations on rational numbers • solve rational equations, including those arising from word problems • work with radical equations and identify multiple solutions and extraneous solutions • graph and solve systems of equations including linear and quadratic equations • extend knowledge of operations with polynomials to the set of complex numbers and find complex solutions. • use the Unit Circle to make connections between trigonometric ratios and trigonometric functions and their properties • understand radian measure • sketch and graph trigonometric functions by identifying key values of these functions as reference points • identify key properties of trigonometric functions and their graphs and justify trigonometric identities such as $\sin(\pi-x) = \sin(x)$ 	<p>(Visible Learning for Mathematics, pp 119-123)</p> <ul style="list-style-type: none"> • Prior knowledge arises in analysis of research as the most influential factor in learning mathematics (Visible Learning for Teachers; pp 41-59). From all the courses in high school, this is especially an important idea for Algebra 2, where all prior learning comes together to shift (for once and all) student’s thinking towards a mathematical perspective where relations, representing relations and generalization of operations are the central focus (Encyclopedia of Mathematical Education; pp 27 32). Therefore, it is of paramount importance that teachers of Algebra 2 engage students' prior knowledge in building their mathematical knowledge. Some strategies to achieve this approach can be: <ul style="list-style-type: none"> ○ provide anticipatory guides when introducing units that elicit prior knowledge of concept and language ○ encourage students to construct semantic maps that combines conceptual knowledge and language ○ include text-sets at various reading levels and media that evokes and consolidate prior knowledge ○ engage with KWL routines that invites students to make connections between what they already know and what they would like to know ○ engage students with structured brainstorming activities in small groups (Academic Literacy in the Academic Disciplines; pp 1-33 and 69-90). <p>Digital Tools & Resources</p> <ul style="list-style-type: none"> • Desmos: this tool is an online dynamic geometry software that includes graphic and scientific calculator and featured activities connected to linear, quadratic and exponential functions, inequalities, geometric transformations and more. (click here) • Padlet: Makes the remote experience come alive with various gallery walls to choose from such as timeline, map, shelves, stream in order to display student work, build on or connect ideas, facilitate a conversations and give feedback (click here) • PBS Learning Media: high-quality resources such as short videos(including videos of real world applications of mathematics from NOVA), lessons and interactive games organized by math topics (click here) • The 3-Act Problems: Dan Meyer’s has created a spreadsheet listing “Three Acts Tasks”: (1) Act One introduces a central conflict of the story/task clearly, visually, viscerally using as few words as possible. (2) In Act Two the student (protagonist of the story) overcomes obstacles, looks for resources, and develops new tools. (3) In Act Three, the student resolves the conflict and set up a sequel/extension. (Click here)
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Functions -Linear,

 Quadratic, and

 Exponential

 Models

- understand the effect of the parameters of trigonometric functions [e.g., in $y = \sin(w(x-h) + k)$; parameters $A, w, h,$] and use these parameters to graph them and solve real-world problems involving periodic motion and sinusoidal functions.
- recognize, manipulate and write equivalent trigonometric expressions by extending the concept of identity element to trigonometry functions
- use exponential functions to model real-world scenarios involving exponential growth and decay.
- use properties of exponents to (1) work with and simplify operations on very large and very small numbers, including numbers expressed in scientific notation and to (2) rewrite expressions involving radicals and rational exponents
- extend understanding of integer exponents to rational exponents, and use laws of exponents to consider the values and meaning of irrational exponents
- use the properties of exponents to write equivalent expressions involving rational exponents (i.e., radicals) and continue to extend their understanding of integer exponent to include all real numbers (i.e., irrational exponents).
- apply their deeper understanding of exponential functions to more complex real-world problems.